1 Introduction

Diagrams are a particularly useful type of sign, in that they can be an aid to cognitive processes (Clark and Chalmers 1998, Zhang and Norman 1994). A deeper understanding of diagrams, for example from a semiotic perspective, has potential for large benefits in how effective diagrams are at supporting specific tasks. This paper is based on a Peircian notion of signification[1]. On the triadic relation, Peirce notes that "If we say it is three subjects, we take at the outset an incomplete view of it" (CP1.471). With this in mind, we utilise a morphic view of the triadic relation, by looking at the mappings between the subjects rather than the subjects themselves, in order to gain new insights into the quality of diagrams. Our contributions are:

– To provide a narrative for the historical evolution of the study of diagrams
– To utilise a morphic view of the Peircian triadic model, in order to derive quality measures for diagrammatic signs

2 Historical Neglect of Diagram Semiotics

2.1 What is a Diagram? Peircian Perspectives

In Peircian semiotics, diagrams can be found within the world of iconic sinsigns, as "a representamen which is predominantly an icon of relations" (CP4.418). For Peirce, iconic sinsigns then include portraits, models, diagrams and maps. Recall also Peirce's definitions used in his triadic model of signification:

> "A representation is that character of a thing by virtue of which, for the production of a certain mental effect, it may stand in place of another thing. The thing having this character I term a *representamen*, the mental effect, or thought, its *interpretant*, the thing for which it stands, its *object*." (CP1.564)

Building on Peirce's definitions to allow greater specificity, a diagrammatic representamen is the substance of an icon of relation (often the substance is "lines on paper" and referred to as a "diagram"). Given the anchoring of this paper in Peircian semiotics, it seems prudent to examine Peirce's thoughts on diagrams. As a mathematician, these naturally focused on geometry, logic and Euler dia-

1 Throughout this paper, we follow the usual volume.paragraph citation format for Peirce's Collected Papers, using as our text Peirce and Moore (1998).

grams. In his thoughts on mental diagrams, Peirce asserted that all mathematical thought is diagrammatic:

> "For mathematical reasoning consists in constructing a diagram according to a general precept, in observing certain relations between parts of that diagram not explicitly required by the precept, showing that these relations will hold for all such diagrams, and in formulating this conclusion in general terms. All valid necessary reasoning is in fact thus diagrammatic. This, however, is far from being obviously true. There was nothing to draw the attention of the early reasoners to the need of a diagram in such reasoning." (CP1.54)

This seems plausible, if we use the definition of a diagram as an icon of relations, since mathematics can be seen as primarily relational in its abstraction. Further, for Peirce mental diagrams have claims on deduction itself:

> "Deduction is that mode of reasoning which examines the state of things asserted in the premisses, forms a diagram of that state of things, perceives in the parts of that diagram relations not explicitly mentioned in the premisses, satisfies itself by mental experiments upon the diagram that these relations would always subsist, or at least would do so in a certain proportion of cases, and concludes their necessary, or probable, truth." (CP1.66)

In terms of diagrams in their specific instantiations, describing a particular situation outside mathematics, Peirce is relatively quiet. In terms of physical manifestations, regarding inference over existential graphs, he says:

> "What we have to do, therefore, is to form a perfectly consistent method of expressing any assertion diagrammatically. The diagram must then evidently be something that we can see and contemplate. Now what we see appears spread out as upon a sheet. Consequently our diagram must be drawn upon a sheet. We must appropriate a sheet to the purpose, and the diagram drawn or written on the sheet is to express an assertion. We can, then, approximately call this sheet our sheet of assertion. The entire graph, or all that is drawn on the sheet, is to express a proposition, which the act of writing is to assert." (CP4.430)

In this context, he also suggests that "A diagram ought to be as iconic as possible; that is, it should represent relations by visible relations analogous to them." (CP4.433). Beyond Peirce, the emphasis in definitions of a diagram are various:
- "An illustrative figure which, without representing the exact appearance of an object, gives an outline or general scheme of it, so as to exhibit the shape and relations of its various parts." Oxford English Dictionary (Oxford University Press 2018)
- "The diagrams, then, are maps of thought, which may be used "to stick pins into" in order to mark anticipated changes." (Kiryushchenko 2015: 117)

– "Diagrams are usually simplified figures, caricatures in a way, intended to convey essential meanings. Diagrams by their very nature do not pretend to be naturalistic; they seek to represent whatever their author regards as the salient features of the subject." (Hall 1996: 9)
– "Diagrammatic form itself provides the facility to describe whole processes and structures, often at levels of great complexity in one representation." (Scaife and Rogers 1996: 1)

It will not escape the reader that, whilst compatible, even within Peirce's own work these definitions are not identical. The line to tread in defining a diagram is perhaps not as obvious as it may seem. Indeed, "landscape art should be considered a branch of topographical illustration, and thus may be classified as scientific illustration" (Topper 1996). By restricting our study to diagrammatic and schematic representations, we are spared this conflict and are able to research with depth. As such, we move forward with the Oxford English Dictionary definition, a restriction of scope from Peirce's thinking.

2.2 Why Diagrams Matter

Diagrams can be useful aids for describing, interpreting and reasoning about systems. Given the above comments by Peirce, it seems clear he would not be surprised by the quantity of praise for diagrams since his death. Tylén et al. (2014) concisely summarise that diagrams:
– Are external representational support to cognitive processes (Clark and Chalmers 1998).
– Make abstract properties and relations accessible (Hutchins 1995).
– Can be in a public space, therefore enabling collective and temporally distributed forms of thinking (Peirce and Moore 1998).
– Are manipulated in order to profile known information in an optimal fashion.

Further commentators have praised diagrams for controlling search space (Sloman 1984), their explicit spatial relation advantage (Karaca 2012) and explanatory value (Burnston 2016). Stenning and Oberlander (1995) state that the power of visual representations is in the omission of information, limiting abstraction to aid "processibility". In a similar vein, Levesque (1989) argues for simplicity, that inferential and computational tractability is maintained by minimising the number of cases that must be computed over. Shimojima's (2015) work on "free rides" includes a number of examples, the core concept of which is that by establishing one relationship in a diagram there is also established a relationship to all objects within the diagram. This leads to well grounded claims for diagrams aiding inference and consistency checking, at the expense of potential over-specificity. To summarise these crucial works, some of the key properties of a diagram are:

1. Being an external support to cognitive process, allowing for reduced utilisation of memory
2. Making abstract properties and relations accessible
3. Making topics simpler, leading to reduced search space and fewer cases to be computed over, by including minimal salient information
4. Being public, allowing for articulation of concepts and distributed thinking
5. Facilitating perceptual "free rides" in inference

2.3 A Brief History of Schematic Diagrams

Diagrams in the modern sense are quite new. Whilst some of da Vinci's illustrations perhaps have the abstraction to be considered diagrams, there is a technological piece missing. This is described in Lawrence's wonderful "History of Descriptive Geometry in England" (Lawrence 2003), relevant aspects of which are drawing upon and summarised throughout this section. Orthographic projection, the technical key to mechanical diagrams, was invented by mathematician and draughtsman Gaspar Monge in Napoleon's French military, eventually published in his "Géométrie descriptive" (Monge 1798). Orthographic projection allows three dimensions to be reduced to two, leading to the ability to describe those dimensions with simplicity and accuracy. This traveled to the US via West Point, but was not propagated in Britain, according to Lawrence perhaps due to the war with France, and the idea originating from a Republican educationalist.

In Britain, Nicholson's Parallel Oblique Projection achieved similar objectives, in appropriately representing a 3D object in a 2D medium. Quoting Lawrence's (2003: 1274) account of Nicholson: "In 1794 I first attempted the Orthographical Projection of objects in any given position to the plane of projection; and, by means of a profile, succeeded in describing the iconography and elevation of a rectangular parallelpipedon: this was published in vol. ii of the Principles of Architecture (1822)". Farish, a professor at Cambridge, had educational mechanical models which he made of smaller components, and he devised a way of graphically describing these to communicate how his mechanical models should be assembled in advance of lectures. He called these graphical communications "isometrical perspective". This was disseminated through the Mechanics Institute as the "British System of Projection" (Lawrence 2003).

To illustrate this historical development timeline, in 1824 in order to articulate his heat-engine concept Carnot included only a single diagram, a high level diagram of a piston without connections (Boon and Knuuttila 2009). In 1840, Babbage's Analytical Engine theoretical designs (Science Museum Group 2019) include plans that could be described as diagrammatic in that they outline key attributes without excessive redundant flourishes. The theoretical nature of Babbage's work makes this abstraction understandable and natural, and indeed in this case the nature of the diagram is not so much iconic, but rather indexical, in that a change to the diagrammatic representamen directly changes the nature of the represented hypothetical object.

"It seems fairly certain that Marc [Isambard Brunel]'s drawings of his block-making machinery [in 1799] made a contribution to British engineering technique much greater than the machines they represented. For it is safe to assume that he had mastered the art of presenting three-dimensional objects in a two-dimensional plane which we now call mechanical drawing." (Rolt 1957: 29-30). These diagrams included projections, but still included what could be seen as *unnecessary flourishes* such as detailing the lever handles and ironwork decorations. Isambard Kingdom Brunel's 1840-1847 plans for the SS Great Britain (Weale 1847) still utilised illustrations without projections, alongside schematics that were more abstract than his father Marc's, and comparable to modern engineering diagrams. After this time, more modern schematics become ubiquitous, if not yet standard. As such, the 1840's appear to be a key turning point for the popular dissemination of this combined *projection-and-abstraction technology*, coinciding with the expansion of the Mechanics Institute but predating Public Libraries. We argue what would be considered today as schematic systems diagrams (distinct from illustrations) have perhaps 200 years of heritage.

2.4 Diagrammatic Lament

Throughout history, diagrams have been a neglected area of academic research. Much attention has been given in semiotics to language, images and icons.

For the semiotics community, perhaps diagrams lack overtly the aesthetic dimensions that makes visual semiotics interesting. It is unfortunate that the Prague Linguistic Circle (1929-1952), with their definition of language as "a system of goal-oriented means of expression" (Steiner 1982: 5) and their visual work did not explore with depth diagrammatic representations. Within the semiotics community, being neither art nor text, diagrams were overlooked by the Tartu-Moscow Circle (1964-present). In modern times, we would argue semiotics is having success down a number of paths including linguistics, education, unification, anthropology and marketing. In considering diagrams as a communication medium, very little has been done. Relating to internal representation and reasoning, related work includes the examination of space on computational speed (Bryant and Tversky 1999), tests on the impact of different visual arrangements on tasks (Kosslyn 1980), and diagrams to support mathematical reasoning and abduction (Hoffmann [2004] and others).

By their nature, scientific illustrations can often be considered as iconic sinsigns, and the set of types of scientific illustrations includes diagrams. This content is nicely summarised by Topper, which we utilise for this paragraph. He cites the crucial nature of scientific illustrations as famously described by Gould (1992: 171), "Scientific illustrations are not frills or summaries; they are foci for modes of thought". From this academic scientific perspective, "Historians of science have shown a little more curiosity, but they too tend to treat scientific pictures only as afterimages of verbal ideas." (Edgerton 1985: 168), and Historian of Science Eugene S. Ferguson (1977: 835) states that visual imagery has been ignored by historians of technology "because its origins lie in art and not in science". To further illustrate the point, his seminal work "The Mind's Eye: Nonverbal Thought in Technology" (1977) there is not a single mention of diagrams or schemata in his historical review. Despite this omission, in modern times diagrams are heavily utilised, particularly in science and engineering. Examples include the periodic table of elements, engineering schematics, and biological life-cycles. Perhaps, where funding exists in these spaces, it is invested in direct technical advancements rather than improving the effectiveness of communication, such as communication through diagrammatic representations.

In more recent times, diagrams are becoming more popular as a research topic. Since 2000 there has been a conference for a diagrammatic interest group, the biannual "Diagrams: International Conference on Theory and Application of Diagrams" which publishes proceedings on philosophy and psychology of diagrams in "Diagrammatic Representation and Inference". The absence of a diagram focused conference until this recent time is indicative of the attention this crucial form of visual communication has received. Shimojima (2015) signposts three as yet un-investigated cognitive potentials of diagrams in envisaging, aspect shifting, and transfer of spatial analysis. These topics are at the core of diagrams as an aid to reasoning. Work such as this, together with the adoption of visual languages for computing, may inspire an increasing rise in popularity of the discipline. We continue to examine the semiotic foundations of diagrams.

3 Diagrammatic Foundations

Ontologies and taxonomies provide language and formalism to be able to describe the diagrams as objects themselves in a rigorous way. Utilisation of the taxonomy found in Engelhardt's thesis (2002) allows fast progress, a choice made to allow exploration of the area without the interlinked complexity of an ontology, nor requiring firmly setting ontological classes.

The application of Category theory, by way of morphisms, to this formalised diagram object allows for analysis of the properties of these morphisms, reorienting discussion from the traditional emphasis of Peircian semiotics, as performed by Vickers et al. (2013) in the specific case of data visualisation. Other conceptually related work includes the definition of "semiotic morphism" *between* sign systems (Goguen 1999, Goguen and Harrell 2005).

For Peirce, a diagram type, apart from its factual individuality, is a (Rhematic) Iconic Legisign. An individual diagram can be considered a Rhematic Iconic Sinsign. For our purposes, it is important that the context changes the act of signification. As Peirce puts it:

> "Icons are so completely substituted for their objects as hardly to be distinguished from them. Such are the diagrams of geometry. A diagram, indeed, so far as it has a general signification, is not a pure icon; but in the middle part of our reasonings we forget that abstractness in great measure, and the diagram is for us the very thing. So in contemplating a painting, there is a moment when we lose the consciousness that it is not the thing, the distinction of the real and the copy disappears, and it is for the moment a pure dream–not any particular existence, and yet not general. At that moment we are contemplating an icon." (CP3.362)

Peirce's focus here is in geometrical diagrams, and highlights the utility of the diagram as a device to hold abstraction. It is instructive to consider different types of diagrammatic representamen, and in particular their content.

3.1 Diagrammatic Structuralism

Engelhardt's thesis examined graphics, including diagrams, and provided a framework for the description (but not creation or utilisation) of these. This work is abbreviated and modernised in Engelhardt (2006) (using trees of objects and sub-objects) and later Engelhardt and Richards (2018) (with a comprehensive listing). Engelhardt proposes the following meta-taxonomy:

– Signs (the components of a diagram): 1 Basic graphic vocabulary, 2 Conventional elements, 3 Pictorial abstraction

- Graphic structure of a diagram: 4 Graphic structure
- Meaning: 5 Mode of correspondence, 6 The represented information
- Context-related aspects: 7 Task and interaction, 8 Cognitive processes, 9 Social context

This taxonomy captures a variety of object-attribute properties, which can be considered semantic, syntactic and pragmatic in nature.

3.2 Diagram Complexity

The number of semantic objects includes both the objects themselves and the relationship between them (though often in diagrammatic representamen, this can be considered a syntactic encoding of a semantic property of the underlying overall object). A commonplace example is a diagram (or map) of a rail network, which prioritises encoding relationships. The important part of this example is that a rail map includes minimal semantic objects, excluding for example time between stations or physical distance, both of which might be helpful to the diagram user. We do not say we make a diagram of a set of stations and their relationships, but rather we make a diagram (or map) of the rail network. The stations are a subset of the rail network, though we would argue that it is often the relationship between these sub-signs which is the key to utility (for example, to navigate from station A to station B).

Perhaps the simplest way to quantify diagram complexity is to count features (semantic and syntactic, excluding pragmatic as they assist cognitive process). We can list and enumerate each Sign and Graphic Structure for a particular instance of a diagram.

4 Morphism properties as quality measures

4.1 A Morphic Perspective

This section contains the substance of the paper, by facilitating quantification of the signification process. In both Peircian and Saussurian (De Saussure 2011) semiotic thinking, the model of thought and representation is oriented around the object and its representation. In a mathematical formalism called “Category theory”, the objects are present, but more important are the *morphisms* between *objects* (Eilenberg and MacLane 1945). Category theory is a mathematical foundation for describing general abstract structures. It includes categories, objects, morphisms, functors, and natural transformations. A *category* is a collection of morphisms. Morphisms between categories are called *functors*, and morphisms between functors are called *natural transformations*.

This *morphic paradigm* allows us to consider the Peircian triadic relation in a new way. Category theory has previously been applied to Visual Semiotics. It is a suitable framework, since it is abstract, and requires simply the existence of objects and relations between them. For the Peircian triadic relation, Vickers et al. (2013) identified some assumptions necessary for the mathematical morphism properties of commutativity, identity and associativity for this visualisation to be considered a Category, and engagingly applied this to "chart junk".

Caterina and Gangle (2016) formalise mathematically the use of Category theory within the Peircian triad, with additional commentary and mathematical framework for icons, allowing for additional insight and precision into iconicity. In the abstraction of our paper, their semiotic morphism would be a functor from the *Triadic category* to itself.

In the concrete instance of an information system, Vickers et al. (2013) captured its objects and morphisms. The existence, even if not explicitly defined, of an Ontology for each category-theoretic object allows us to represent the objects of each morphism as "sets" in the mathematical sense.

Note that the properties of these morphisms determine operations such as generalisation, abstraction, amplification etc ("operations" noted by Peirce) (see CP4.505-4.509, and Wason and Johnson Laird [1972]). It is in these properties of the morphisms that we hope to analyse the quality properties of the diagrammatic representation.

Figure 1 shows the familiar Peircian triadic model, with a reorientation to the implicit *representational morphisms* that occur in the Peircian semiotic process. Note that we have included directional arrows for clarity of definitions, and in an attempt to understand one aspect of the semiotic process. The encoding morphism is about modelling the real world, in doing so "parsing" or instantiating the real world into a set of visual semantic and syntactic actualities. We consider the encoding morphism as a function of the sender, rather than the receiver.

In order for the diagram to be considered a representation (or more strongly as a sign), there should be a relationship between "real world" and "diagrammatic representation". Each step of this mapping sequence can lead to error or inefficiency. Unlike an Information System, where the preservation and completeness of information is desirable, in a diagrammatic representation it may be desirable to lose information in the *encoding morphism* order to make the *pragmatic morphism* more cognitively computationally efficient. Stenning and

Oberlander (1995) assert that graphical representations limit abstraction and in doing so support better “processibility”. As such, this morphism of diagram creation is not automatically invertible and the category diagram in Figure 1 does not automatically commute. Vickers et al. (2013) explored the restrictions required for different properties of these morphisms. In keeping with our understanding of semiotic theory, the perceptual morphism is not a naïve view on the real world without mediation, but is an extremely complex morphism, in itself a function of the receiver’s context. The pragmatic morphism is concerned with the usage of the diagram in a particular task or set of tasks. It is dependent on the receiver.

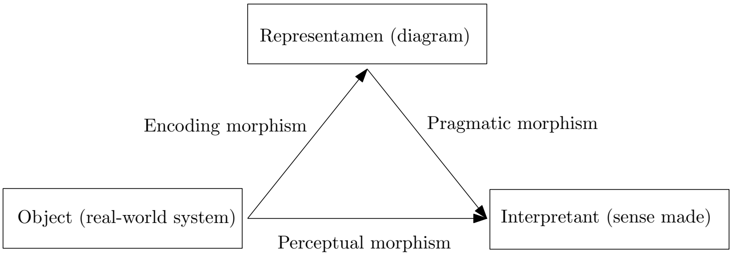

Fig. 1: Peirce’s Triadic Model with Representational Morphisms

4.2 Encoding Morphism Quality

To discuss the encoding morphism, it is helpful to have a specific application in mind. In computer science, diagrams are often used to signify system designs. Whilst some aspects of these diagrams may be prevalent and recurrent (such as the use of arrows, and the spatial encoding of temporal dimension), common practice does not employ any explicit formalism. Even for the case of Unified Modeling Language (UML), a formally defined and ontologically grounded diagrammatic language for modeling, there is no convention around use of colour, size and location, and these properties may or may not convey meaning (Moody 2009). Recently, it has been shown that the diagrammatic representations employed within a sub-field of computer science are extremely heterogeneous (Marshall and Freitas 2018). This example indicates that we may be unable to usefully consider “types” of diagrams at a pure representational level, except in cases where more fully defined frameworks are both available and utilised.

Wand and Wang (1996) ground "data quality" in an ontological framework between Real-World states and the Information System states representing this. Adapting this, in Table 1 we propose intrinsic quality dimensions for the transformation from a *Real-world system* to a *Diagrammatic representamen* (i.e. the Encoding Morphism). These remain relevant in this new context, as they are derived directly from the morphism properties (for example if there are objects in the domain for which there is no mapping that would lead to incompleteness). These are at the morphism level, and for specific instances this is instructive. For a given morphism, unlike with data quality, we would not wish for the representation to be complete due to the unnecessary semantic content. Unambiguity, Meaningfulness and Correctness are still usefully considered at this individual level.

Table 1: Intrinsic diagram quality dimensions

Encoding Morphism Quality Dimension	Definition	Deficiency
Complete	The ability for the diagram states to represent all System states	There exist certain System states that are not represented
Unambiguous	Diagram states maps back to precisely one System state	It is possible to map back to multiple system states
Meaningful	It is always possible to map a given diagram state to a System state	The diagram represents an impossible System state
Correct	Each diagram state maps to the correct System state	The diagram state is mapped to the wrong System state

The apprehension principle (Tversky et al. 2002) suggests that external representations should be readily and accurately perceived and conceived. As stated, this principle would be a property of the pragmatic morphism. In practice, this would involve omitting (in terms of usage, unnecessary) details from the encoding morphism. Through a taxonomical representation of the diagram, we can explicitly define and reduce these. For example, it is clear that colour, outline, and position can be used for grouping objects, so if multiple groupings are required it could be advisable to use these in a coordinated fashion, for example aligning objects which are coloured similarly. Conversely if multiple groupings are not required to transmit the required semiotic information from the Interpretant, then perhaps entire diagrammatic properties (such as colour) should also be omitted. However at the structural level, we agree with Boon and Knuuttila (2009) that a representation should be structurally isomorphic with that which it represents. For our purposes, the encoding morphism should result in a diagrammatic representamen structurally isomorphic to the object. This is naturally in keeping with the Peircian definition of a diagram.

In this work we are concerned with the quality of this representation, which involves understanding of the diagram's context, including characteristics of those who will use it, and how it will be used.

4.3 Pragmatic Morphism Quality

4.3.1 Utility of diagrams to accomplish tasks

It is not sufficient for diagrams to be complete, unambiguous, meaningful and correct. Diagrams must also be useful (a cognitive aid to relevant tasks) and efficient (information is easy and reliable to access correctly). This section covers "Pragmatism", the interpretation of diagrams.

From the perspective of the diagram author, there should also be consideration of *intended semantic payload* of the diagram. When considering the effectiveness of diagrams as a communication medium, this and "coding" would be highly relevant (see Cobley [2013]). However, we are considering diagrams as a tool to support tasks. Through this lens, the intended usage is not as relevant as the task to which it is applied in reality.

Whilst this may seem a harsh standard to impose, it is necessary to assess against the actual usage, rather than the author's thoughts on how the diagram will be used. The usage is the test that the diagram will be subjected to, not simply the hypothetical one for which they were designed. By invoking a particular context, we are transforming from an iconic sinsign (representament, object, interpretant: secondness, firstness and firstness) to a dicent indexical(/iconic) sinsign (secondness, secondness and secondness).

As put by Short (1982: 287), "Since goals can often be obtained only by taking risks, there will be some fallible grounds of interpretation. This makes it possible for there to be false or misleading signs. These have significance and interpretability, yet what they signify is not.". In our model of the sign process, this thought spans both encoding and pragmatic morphisms, and we are able to pull apart (but not entirely separate) concepts to make salient the difference in improper encoding of (false) signs by the author, from the pragmatic interpretation of (misleading) signs by the interpretant user. Note that our reorientation does no more to separate the concept of a sign into its triadic components of Object, Representamen and Interpretant, but in changing the way we view these we gain different insights into the signification process.

4.3.2 Utility of other types of iconic sinsign

Apart from diagrams, other iconic sinsigns, for Peirce, include portraits, maps and models. For portraits, the required outcome is less obvious. We argue that as such this pragmatic morphism need not be discussed in depth, and in this case the encoding morphism is much more important. For some types of iconic sinsigns, such as those used in marketing, there is perhaps a human response which could be optimised for, such as brand association, but we omit this as being more appropriate for a separate discussion on qualisigns.

For maps and models, as with diagrams, there is a task at hand (often that of navigation from a point to another, or planning a route), so the pragmatic morphism is important to understand, and can in principle be measured in similar ways to the diagrammatic focus here. In an experiment using time-taken and error-rate of geographically-based managerial tasks, Smelcer and Carmel (1997) found maps more effective than tables.

Relevance and consensus (or standards) also apply for cognitive efficiency of the diagrams themselves. Diagrams facilitate “free rides”, as a result of consequential reasoning (Shimojima 2015). Shimojima continues to highlight electrical engineers adopting semantic techniques beyond basic conventions, such as spatial closeness for conceptual proximity. This can of course back-fire if diagrams are created without thought, as Marks and Reiter (1990) demonstrate.

4.3.3 Desirable pragmatic morphism properties

There is an absolute truth for the representamen that is the diagram, which could be captured as a taxonomy. Similarly, the real world knowledge activity is modeled as having an absolute truth. At very least we have an “island of attempted rationality” (Brier 2008: 106), provided we reject solipsism or highly individualistic views of knowledge. There is also a perceived truth in the human mind. Even in the simple case of Boolean logic, perception morphisms can be considered nonmonotonic, and as such do not preserve relations of the real world domain (Arzi-Gonczarowski and Lehmann 1998). Whilst this is a complex topic, we can suggest desirable properties of this morphism:

- The encoded semiotic content is pertinent for solving the required problem.
- The proportion of semiotic content lost between diagram and mind should be minimal.
- There will be impact on user associative memory, spatial ability and visual memory (Chen and Yu 2000). For example, Cognitive Load Theory suggests a two to four operational element limit (Van Merrienboer and Sweller 2005), further restricting with Miller's (1956) magic 7+/-2 for Short Term Memory.
- Optimising for Coherence would encourage identifying and removing redundant elements, as has been shown for educational outcomes (Mayer and Moreno 2003).
- It may be that dual coding (simultaneous processing of the two modalities of linguistic and non-linguistic elements) has an effect on memory and processability (Paivio 1991).
- The user's *Skill* (long-term experience leading to unconscious reaction), *Rules* (predefined process reducing cognitive load), and *Knowledge* (highest cognitive load, for novel problems) are likely important (Cantu et al. 2017).
- The diagram should appear aesthetically appealing, with users wanting to use it.
- Other cognitive dimensions exist, such as distiguishability of elements, a manageable number of symbols, and hidden dependencies (Green and Petre 1996).

As can be seen from this small subset of considerations, the complex "pragmatic morphism" can also facilitate any of the Encoding quality defects: meaninglessness, ambiguity, incompleteness and inaccuracy. In our pulling-apart of the sign process, there are more natural pragmatic quality dimensions based on the list above, not based on the Foundation Ontology of the representamen but more on psychological concepts.

Table 2 shows some core pragmatic morphism quality dimensions for diagrams. Note that "Simplifying" and "Essential" to some degree counterbalance one-another, reflecting the tension between wishing to include content whilst being easily understood. Thorough quantification of this perceptual process is beyond the scope of this paper, and arguably beyond modern psychology. Further, a *simple quality measurement* is required in order for the measure to be useful beyond a theoretical concept.

We can consider solely whether the diagram is cognitively efficient. Perhaps the simplest useful measure is *Mean Correct Response Time* when answering

Table 2: Pragmatic diagram quality dimensions

Task-based Pragmatic Morphism Quality Dimension	**Definition**	**Deficiency**
Simplifying	The information preserved by the pragmatic morphism is lower than the perceptual morphism	Redundant concepts are expressed and it is difficult to navigate
Essential	The diagram supports the execution of specific useful tasks	The diagram does not contain the semantic content required
Integrable	The diagram supports integration of concepts into existing mental models	There is a high conceptual or cognitive barrier to entry, notation is confusing, ambiguous or unintuitive
Inferable	The diagram supports materialisation of new concepts	Performing abduction, deduction and induction is difficult, there is no space to add or manipulate concepts
Aesthetic	The diagram is pleasing to look at	Users do not enjoy looking at or using the diagram

a problem related to a particular task (for an example, see Ziemkiewicz and Kosara (2008)). Following Hoffman and Schraw (2010), we can consider modeling cognitive efficiency as "Performance" (proportion of correct attempts at a task) vs "Effort" (variously measured, typically time taken but could also be physical measures such as cerebral glucose usage). Based on the relationship between performance and effort, Hoffman and Schraw suggest three different options in terms of modelling cognitive efficiency:

– Deviation model (a standardised difference between performance and effort)
– Likelihood model (correct responses per unit effort)
– Conditional likelihood models (relationship between performance and effort across a set of different problem solving tasks, to assess consistency)

For comparative purposes, we advocate the use of likelihood, the ratio of correct per unit effort, as the overall pragmatic morphism quality measure. This is a good measure since, as a ratio, it is scale invariant and is more resilient to experimental differences. It is also more straightforward to measure than conditional likelihood. To summarise, we suggest creating a single, common, utility-appropriate task with known answers, and measure as follows:

– To quantify *effort* of using diagrams, we will take "mean response time" as a proxy.
– To quantify *performance* of using diagrams through correct response rate = $\frac{\text{count of success}}{\text{total attempts}}$.
– The overall quality of the pragmatic morphism: $\frac{\text{performance}}{\text{effort}} = \frac{\text{correct response rate}}{\text{mean response time}}$.

The necessary restriction "mean response time >0" and "total attempts >0" make these measures well-defined. Note that effort $\in \mathbb{R}$ and performance $\in [0, 1]$, so overall quality $\in [0, 1]$.

There can be utility-critical semantic content omitted by the encoding morphism which would negatively impact the performance metric. In either case, the Diagrammatic Representation object itself would warrant further investigation. We cannot fully separate object, representamen and interpretant, and nor can we fully separate their morphisms.

4.3.4 Aesthetics

As Rudner (1951: 67) puts it, "The history of aesthetics is replete with schemes of classification for theories of aesthetics". Dimensions of aesthetics have been modeled in a qualitative fashion in a variety of ways. To take two relatively recent examples, we have partitioning into two of "interest" and "pleasure" (Graf and Landwehr 2015), or into six primitives: colour, form, spatial organisation, motion, depth and human body (principle-axes choice) (Peters 2007). Rather than explore these or other classifications, we necessarily focus on measurable work. According to Rudner, Charles Morris considers an aesthetic sign as a particular type of (iconic) sign, and defines "aesthetic analysis" as a particular kind of sign analysis. Rudner (1951: 71) states "The differentia of an aesthetic value property is that it is immediately or immanently consummatory".

For measures of aesthetics, the seminal work was by Birkhoff, who suggested aesthetics is about "harmonious interrelation within the object" (Birkhoff 1933: 4). This led him to define his aesthetics measure as order divided by object complexity, for some objects such as shapes and vases. He provides guidelines for measuring both order and complexity, the former as symmetry with some adjustments, and the latter as the number of lines. Filonik and Baur (2009) summarise the state of measurement of aesthetics, which helped inform this section.

For patterns, Klinger and Salingaros (2000) algorithmically and iteratively examine 2x2 arrays, and their symmetry, to capture information and symmetries to allow derivation of structure and randomness (which they keep as separate axes).

For digital images, Rigau et al. (2008) apply Shannon entropy to the RGB palette of pixels, to measure order. Together with JPEG compression as an approximation of Kolmogorov complexity, they are able to quantify in a Birkhoffian order-to-complexity manner. Nešetřil's (2005) combinatorial entropy takes a random infinite line through the image and counts intersections, and suggests that a harmonious image has similar combinatorial entropy for its meaningful sub-parts as for the global, capturing whether the image is harmonious and balanced.

In our case of diagrams, further experimental work is required to assess which of these measures, if any, is correlated with aesthetic feeling. The intuitive answer for diagrams seems to be less on symmetry, so we propose using combinatorial entropy for now.

4.4 Summarised Diagram Quality Measures

In total, we propose the following diagram quality measures:

1. Diagram complexity = Count of diagrammatic elements
2. Diagram pragmatic utility quality = $\frac{\text{correct response rate}}{\text{mean response time}}$ (for a specific task)
3. Diagram pragmatic aesthetic quality = Nešetřil's Combinatorial Entropy

4. Diagram semiotic quality = Correct, Meaningful, and Unambiguous mapping expert assessment
5. Diagram feedback = qualitative desirability feedback from users

These measures, taken together, provide a holistic view of the quality of a diagram, providing insight into content, utility and usability. In doing so, these measures facilitate the quantitative comparison of diagrams.

5 Conclusion

In this paper, we have applied Category theory to semiotics in order to gain fresh insight into diagrammatic representations. We have utilised a taxonomy in establishing a fundamental set of diagrammatic features in order to encode semiotic properties. We have used the principles of Category theory to capture morphisms relevant for diagrammatic signs. The unification of taxonomy and Category theory is beneficial and unusual, due to their differing abstraction levels. Applying this theoretical groundwork, we identified useful properties of these morphisms, and used this to assemble a framework by which to assess diagram quality.

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